Unit 4 - Uniform Circular Motion

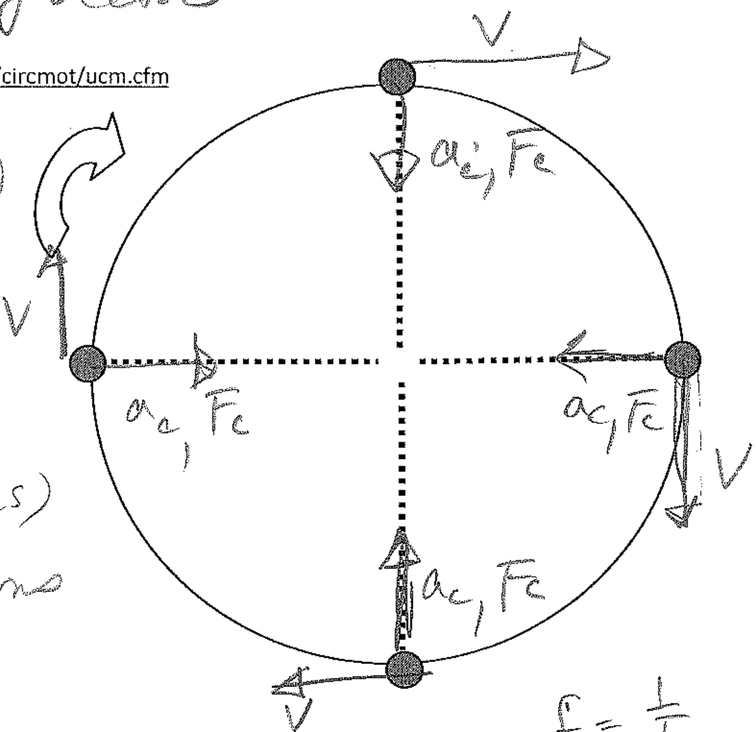
Concept question: Jillian is in a car traveling at a constant speed of 10 m/s in the school parking lot making a circular path. Is her car accelerating?

Yes, accel means change in velocity. Change in direction is a change in velocity vector

Uniform Circular Motion is:

see the following: http://www.physicsclassroom.com/mmedia/circmot/ucm.cfm
http://www.physicsclassroom.com/mmedia/circmot/rht.cfm
http://www.physicsclassroom.com/mmedia/circmot/cf.cfm

When an object is forced to follow a circular path Δv = having accel!



Vocabulary:

Period: Time for one revolution in circle (s)
T =

Frequency: number of revolutions in one sec (s^{-1} or hz)
f =

Centripetal: center seeking

real world ex: yoyo, rides, CD + DVD, centrifuge, planets, e⁻ around nuc, spin cycle

$f = \frac{1}{T}$

$T = \frac{1}{f}$

Equations:

Linear Motion	Circular Motion
Velocity $v = \frac{\Delta d}{t}$	Circular Velocity $V = \frac{2\pi r}{T}$ (circumference) (time)

Example 1: Trevor swings Mr. Ralbovsky's keys to the auditorium (and all the hidden little rooms) around in UCM by a string with a radius of 0.8 m. If the keys make 20 swings in 10 seconds, what is the speed of the ~~mass~~ keys?

$r = 0.8 m$

$T = \frac{10 s}{20 rev} = 0.5 s$

$f = \frac{20 rev}{10 s} = 2 s^{-1}$ or 2hz

$V = \frac{2\pi r}{T} = \frac{2\pi(0.8 m)}{0.5 s}$

$= 10$ m/s

Linear Motion	Circular Motion
Acceleration $a = \frac{\Delta v}{t}$	Centripetal Acceleration $a_c = \frac{v^2}{r}$ $\quad \frac{\frac{m^2}{s^2}}{m} \times m = \frac{m}{s^2}$ (similar triangle derivation in book)

Example 2: What is the centripetal acceleration of the mass in example 1?

$r = 0.8\ m$

$T = 0.5\ s$

$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{(10\ m/s)^2}{0.8\ m} = \boxed{125\ m/s^2}$

$\frac{2^2\pi^2 r^2}{T^2}$ ... $6r$

Example 3: If Brian is dribbling his soccer ball in a circle with a radius of 40m at 10m/s, what is his acceleration? Directed towards where?

$r = 40\ m$

$v = 10\ m/s$

$a_c = \frac{v^2}{r} = \frac{(10\ m/s)^2}{40\ m} = \boxed{2.5\ m/s^2}$

Example 4: Phil, a future fighter pilot can withstand an acceleration of 8g's before passing out. His plane is traveling at 500 m/s. What is the minimum radius of curvature the plane can perform for the pilot Phil to remain conscious?

$a = 8$ "g's" $= 8(9.8\ m/s^2) = 78.4\ m/s^2$

$v = 500\ m/s$

$r = ?$

$a_c = \frac{v^2}{r}$

$78.4\ m/s^2 = \frac{(500\ m/s)^2}{r}$

$\boxed{r = 3189\ m}$

Linear Motion	Circular Motion
Force $F = ma$	Centripetal Force $F_c = ma_c$ $kg \cdot m/s^2 = N$

← not a new force! $\Sigma F = F_c$

Example 5: A 5 kg mass travels in UCM connected to a string with a radius of 2 m. The mass makes 100 revolutions in 2 minutes. If the string can supply a maximum force of 250 N, will the string break?

$r = 2m$
$m = 5kg$
$T = 0.02 \min \times \frac{60s}{1m} = 1.2s$

$F_c = ma_c$
$F_c = m\frac{v^2}{r}$
$F_c = m\frac{\left(\frac{2\pi r}{T}\right)^2}{r}$

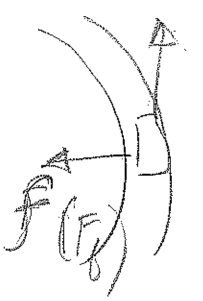
$F_c = 5kg \frac{\left(\frac{2\pi \cdot 2m}{1.2s}\right)^2}{2m}$
$F_c = 275N$
string will break!

Centripetal Force is a force required for circular motion of certain conditions to occur. This "required force" is provided by some ***"real"*** force like:

- Friction
- Gravity
- A string
- Your door when you go around a corner

Example 6: What is the maximum speed a car can travel around a flat corner with a radius of 50.0 m if the road is A) dry? B) wet? μ_s dry=0.900, μ_s wet=0.100 μ_s = car tires!

$r = 50m$



$f = F_c$
$\mu F_N = ma_c$
$\mu mg = m\frac{v^2}{r}$

Dry
$\mu g = \frac{v^2}{r}$
$0.9(9.8 m/s^2) = \frac{v^2}{50m}$
$v = \sqrt{(0.9)(9.8)(50)}$
$v = 21 m/s$ Dry

Wet
$\mu g = \frac{v^2}{r}$
$(0.1)(9.8 m/s^2) = \frac{v^2}{50m}$
$v = \sqrt{(0.1)(9.8)(50)}$
$v = 7 m/s$ Wet

Example 7: What is the minimum coefficient of friction between a car's tires and the road for it to travel in a circle with a radius of 20 m at a speed of 30 m/s?

$\mu_s = ?$

$r = 20\ m$

$v = 30\ m/s$

$f = F_c$

$\mu F_N = m a_c$

$\mu m g = m \frac{v^2}{r}$

$\mu (9.8\ m/s^2) = \frac{(30\ m/s)^2}{20\ m}$

$\mu = 4.6$

not reasonable ∴ can't do curve at that radius @ that speed

Concept Questions:

Explain how circular velocity, centripetal acceleration, and centripetal force are affected by the following changes.

A) Doubling the period of revolution of an object in circular motion.

$V = \frac{2\pi r}{T} = \frac{1 \cdot 1 \cdot 1}{2} = \frac{1}{2}$, $v = \frac{1}{2}$ | $a_c = \frac{V^2}{r} = \frac{(.5)^2}{1} = 0.25$, $a_c = \frac{1}{4}$ | $F_c = m a_c = 1(\frac{1}{4}) = \frac{1}{4}$, $F_c = \frac{1}{4}x$

B) Doubling the mass of an object in circular motion.

$V = \frac{2\pi r}{T}$ = no effect | $a_c = \frac{v^2}{r}$ = no effect | $F_c = m a_c = 2 \cdot 1$, $F_c = 2x$ double

C) Doubling the radius of the circular motion. (keep period the same)

$V = \frac{2\pi r}{T} = \frac{1 \cdot 1 \cdot 2}{1} = 2$, $v = 2x$ | $a_c = \frac{v^2}{r} = \frac{(2)^2}{2} = 2$, $a_c = 2x$ | $F_c = m a_c = 1 \cdot 2 = 2$, $F_c = 2x$

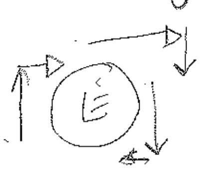
Apparent Weightlessness

Apparent weightlessness is caused by: an object having the same accel in same direction as gravity. i.e. $a = -9.8\ m/s^2$

$F_N - F_g = ma$ $-mg = ma$

Objects that are in orbit are: in constant free fall toward the earth, however they match the curve of the earth as they fall & never crash

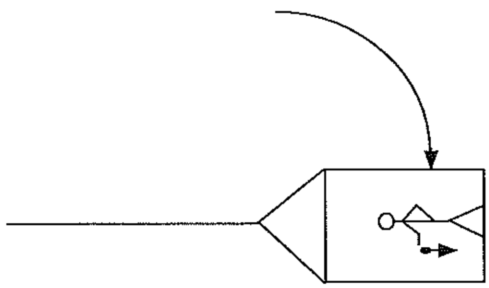
Gravity of Earth provides F_c

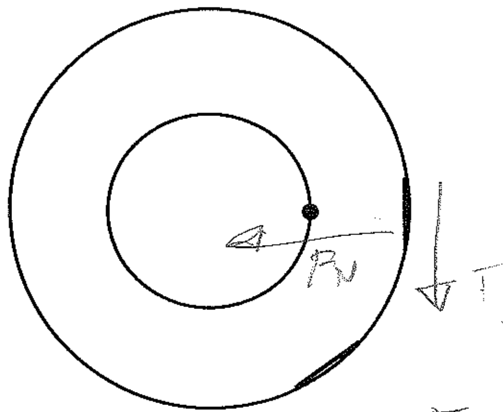


magic speed = ~ 8000 m/s

Artificial Gravity

Imagine a person in a large can at the end of a cable (space station) and swung in a circular orbit.





Path of the ball and the floor.

$F_N = F_c$

The floor of the space station pushes on the feet of a person to cause them to go in circular motion. In other words, the floor provides a centripetal force on the person. The "push" of the floor on the person is like the normal force felt on earth. Therefore, gravity is simulated.

Example 8: A space station has a radius of 1700 m. How fast must the surface of the space station spin to simulate gravity?

$r = 1700 m$

$v = ?$

$F_N = F_c$

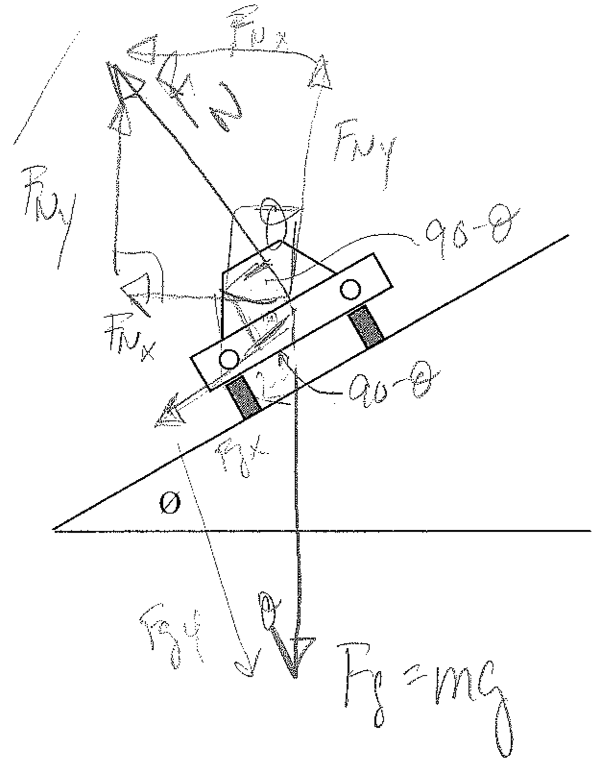
$mg = ma_c$

$g = \frac{v^2}{r}$

$9.8 m/s^2 = \frac{v^2}{1700 m}$

$v = 129 m/s$

Banked Turns



Need a force pointed straight in
|| to the horizontal!
⊥ not F_{gx}!

$F_{gy} = F_N$

F_{Nx} is F_c!

Watch all the angles!

Example 9: Michael is thrilled that he has won a once in a lifetime chance to drive in a NASCAR race. One of the turns at Daytona International Speedway has a radius of 316 m and is banked 31°. What is the maximum speed that Mike can safely travel around the turn?

(Won't be on test!)

$F_C = F_{Nx}$

$\frac{mv^2}{r} = \sin 31° \, F_N$

$F_N = \frac{mv^2}{r(\sin 31)}$ so

need F_N!

$\cos\theta = \frac{F_{Ny}}{F_N}$ = mg or F_g!

$\cos\theta = \frac{mg}{F_N}$ ← hypotenuse

equal

$F_N = \frac{mg}{\cos 31}$

$\frac{mv^2}{r \sin 31} = \frac{mg}{\cos 31}$

$v = \sqrt{\frac{g \cdot r \cdot \sin 31}{\cos 31}}$ (tan!)

$v = \sqrt{g \cdot r \tan 31}$

(9.8)(316)(.6)

= 43 m/s

Satellite Motion

What are the factors that determine the speed of a satellite in orbit?

See the following: http://www.physicsclassroom.com/mmedia/vectors/sat.cfm

$F_g = F_c$

$\frac{G m_1 M_E}{r^2} = \frac{mv^2}{r}$

M_E = Mass earth

Universal

(1) Radius of orbit

– mass of object you wish to orbit

Example 10: The CIA is concerned about the strange experiments that Dan has been doing in the woods behind his house, so they have decided to launch a satellite that will remain over his house at all times. At what altitude does this geosynchronous satellite have to orbit the earth in order for it to remain over Tully?

$T = 24\,hrs = 8.64\times10^4\,s$

$F_g = F_c$

$\frac{G m M_E}{r^2} = \frac{m v^2}{r}$

$r = \frac{G M_E}{v^2}$

$r = \frac{G M_E}{\left(\frac{2\pi r_E}{T_E}\right)^2}$

rearranged (see book p 142)

$r^{\frac{3}{2}} = \frac{T\sqrt{G M_E}}{2\pi}$

$r = 4.23\times10^7\,m$ from earth's center!

$\therefore$ subtract r of earth $(6.4\times10^6\,m) = 3.6\times10^7$ for all geo!

Example 11: What is the speed necessary for a satellite to orbit the earth at a distance of 7×10^8m from the center of the earth? (Mass earth = 6×10^{24}Kg)

$F_g = F_c$

$\frac{G m M_E}{r^2} = \frac{m v^2}{r}$

$v = \sqrt{\frac{G M_E}{r}}$

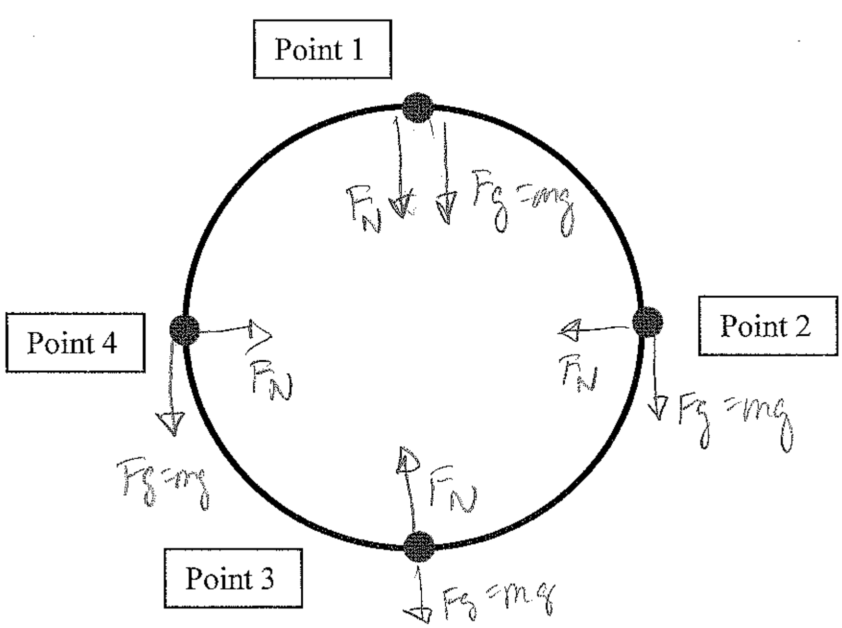
$v = \sqrt{\frac{(6.67\times10^{-11}\,N\cdot m^2/kg^2)(6\times10^{24}\,kg)}{7\times10^8\,m}}$

$v =$

Vertical Circular Motion

Because the motion of a vertical loop is in the same plane as gravitational acceleration, the analysis of the motion is a bit more complicated.

Imagine a mass being whirled in a vertical circle by a string.



Four Critical points in the motion - see the following: http://www.physicsclassroom.com/mmedia/circmot/rcd.cfm

Velocity will change as position changes

Point 1 = top

$F_c = F_N + mg$

$\frac{mv_1^2}{r} = F_{N_1} + mg$

Point 2 (R) side

$F_c = F_N$

$\frac{mv_2^2}{r} = F_{N_2}$

Centripetal force not constant!

Point 3 = bottom

$F_c = F_N - mg$

$\frac{mv_3^2}{r} = F_{N_3} - mg$

Point 4 (L) Side

$F_c = F_N$

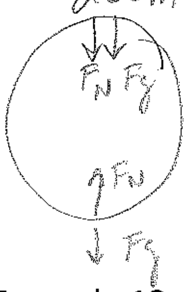
$\frac{mv_4^2}{r} = F_{N_4}$

Example 12: Dave has become a fighter pilot too. He has a mass of 75 kg and is traveling at 112 m/s when he goes into a loop-the-loop with a radius of 200 m. What is his apparent weight: A) at the bottom of the loop B) at the top of the loop. C) How many "g's" is he experiencing at each location?

$m = 75kg$

$v = 112 m/s$

$r = 200m$



(A) $F_c = F_N - F_g$

$F_N = \frac{mv^2}{r} + mg$

$F_N = \frac{75kg(112m/s)^2}{200m} + 75kg(9.8m/s^2)$

$F_N = 5439N$ $\frac{5439N}{75(9.8)N}$ 7.4 "g"

(B) $F_c = F_N + F_g$

$F_N = \frac{mv^2}{r} - mg$

$F_N = \frac{75(112)^2}{200} - 75(9.8)$

$F_N = 3969N$ $\frac{3969}{75(9.8)}$ = 5.4 "g"

Example 13: Hank is on the superman roller coaster at Darien Lake that has just reopened due to technical difficulties and he is worried about the safety so he decides to time the velocity of the train before the vertical loop. What is the minimum velocity that a roller coaster can safely go through a vertical loop of radius 50 m? = top velocity!

Minimum speed means $F_N = 0$!

$r = 50m$

$F_c = F_N + mg$ ($F_N \to 0$)

$\frac{mv^2}{r} = mg$

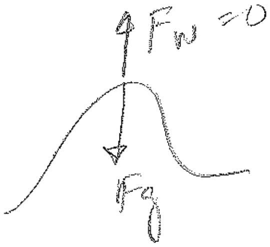
$v = \sqrt{rg}$

$v = \sqrt{(50m)(9.8m/s^2)}$

$v = 22m/s$

Show motorcycles "ball of death"

Example 14: Tirzah is roller blading on Gatehouse Road, racing up one side of the hill and down the other fast enough to lose contact with the ground as she reaches the peak. If the hill makes a circular arc with a radius of 30m, find the minimum speed necessary for her to reach at the top of the hill in order for her to experience that momentary free fall weightlessness.



$r = 30m$

$F_c = F_g$

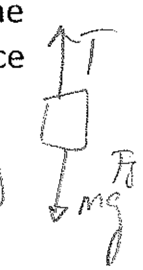
$\frac{mv^2}{r} = mg$

$v = \sqrt{gr}$

$v = \sqrt{(9.8 m/s^2)(30m)}$

$v = 17 m/s$

Example 15: Theresa is hanging motionless with one arm from the top of the uneven bars before she starts her routine. What is the tension that her arms must provide if we assume she has a mass of 40 kg? After she has started, she is doing great big swings around the one bar with one arm, creating a circle with a radius of 1m and a velocity of 2.8 m/s. What force must her arm provide in order for her to maintain that circular motion?

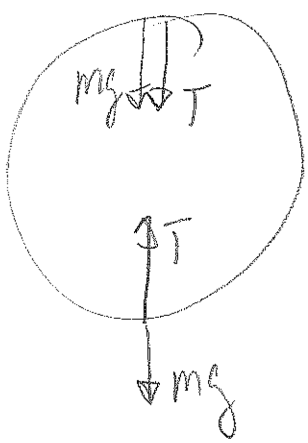


look at top + bottom

no swing $T = mg$

$T = (40kg)(9.8 m/s^2)$

$T = 392N$



$r = 1m$

$v = 2.8 m/s$

Top

$F_c = F_g + T$

$\frac{mv^2}{r} = mg + T$

$T = \frac{mv^2}{r} - mg$

$T = \frac{(40kg)(2.8 m/s)^2}{1m} - 40kg(9.8 m/s^2)$

$T = -78.4N$ (less!)

Bottom

$F_c = T - F_g$

$\frac{mv^2}{r} = T - mg$

$T = \frac{mv^2}{r} + mg$

$T = \frac{(40kg)(2.8 m/s)^2}{1m} + 40kg(9.8 m/s^2)$

$T = 705.6N$

$mg = ma_c$

$392N = 705.6N$

1.8g

Application of Vertical Circles:

Walking and Circular Motion

The swinging motion of your legs when you walk can be modeled approximately as circular motion, and this model can be used to find the maximum speed a person of a given leg length can walk. Humans keep the weight-bearing leg straight while walking. Consequently, as shown in Figure 6.10, each hip in turn moves in a circular arc with a radius R equal to the length of the leg and with the weight-bearing foot at the center of the circle. In the simplest model, we imagine the person's entire mass M to be concentrated near the hip. In this model, when a person walks with speed v, the force required to keep the hips in circular motion is

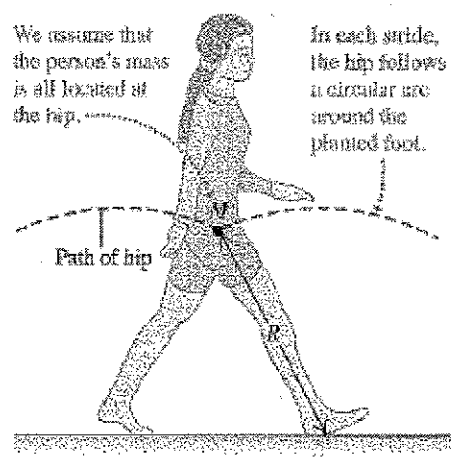
$$F_{rad} = \frac{Mv^2}{R}.$$

Gravity provides this force, so the maximum available force is the person's weight Mg. Thus, in this simple model, the maximum speed v_{max} a person can walk is given by

$$Mg = \frac{M(v_{max})^2}{R}, \qquad \text{or} \qquad v_{max} = \sqrt{gR}.$$

The average adult leg length is about 1 m long; assuming this length, we get

$$v_{max} = \sqrt{(9.8\text{m/s}^2)(1.0\text{ m})} = 3.1\text{ m/s, or } 7.0\text{ mi/h}.$$



▲ **FIGURE 6.10** A simple model of human walking.